Eur. Phys. J. C 18, 369–378 (2000)
Digital Object Identifier (DOI) 10.1007/s100520000516

THE EUROPEAN PHYSICAL JOURNAL C

Multibaryons with strangeness, charm and bottom

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Received: 21 September 1999 / Revised version: 1 February 2000 /
Published online: 27 November 2000 – © Springer-Verlag 2000

Abstract. The spectra of baryonic systems with strangeness, charm and bottom are considered within a "rigid oscillator" version of the bound state soliton model. The static properties of multiskyrmions, of baryon number up to $B = 8$, are calculated using the recently suggested rational map ansaetze as starting field configurations. The property of binding of flavoured mesons by an $SU(2)$ skyrmion is proved rigorously within this model. Binding energy estimates are made of the states with largest isospin which can appear as negatively charged nuclear fragments and for states with zero isospin – fragments of "flavoured" nuclear matter. It is shown that for all types of flavour and for $|F| \leq 2$ the isoscalar baryonic systems have a better chance to be stable against strong and electromagnetic interactions than those with nonzero isospin. Baryonic systems with charm or bottom quantum numbers are found to be bound more than strange baryonic systems.

1 Introduction

The problem of the existence of fragments of baryonic matter with unusual properties, such as their flavour being different from that of u and d quarks, has a rather long history but, so far, has not been resolved. One of the difficulties is connected with the absence of a complete relativistic theory of many-body bound states. In addition to being of general interest, this issue can have important consequences in astrophysics and cosmology. In particular, the formation and subsequent decay of such fragments could be important in the early stages of the evolution of the Universe.

The topological soliton models, and the Skyrme model among them [1], provide a reasonable way to "circumvent" the unsolved questions and to obtain predictions for the spectrum of states possessing strangeness, charm or bottom quantum numbers. The models of this kind are attractive because being based on only a few fundamental principles and basic ingredients they are not only simple but may also well describe various properties of low energy baryons,

The description of skyrmions with large baryon numbers has long been perceived as being difficult because the explicit form of the fields is not known. The recent observation [2] that the fields of the $SU(2)$ skyrmions can be approximated accurately by rational map ansaetze giving the values of masses close to their precise values has considerably simplified the task of such studies. Similar ansaetze have also been recently presented for $SU(N)$ skyrmions (which are not embeddings of $SU(2)$ fields) [3].

In this paper we use the $SU(2)$ rational map ansaetze as the starting points for the calculation of static properties of bound states of skyrmions necessary for their quantization in the $SU(3)$ collective coordinate space. The energy density of the $B = 3$ configuration has tetrahedral symmetry, of $B = 4$ the octahedral (cubic) one [4], of $B = 5$ D_{2d} symmetry, of $B = 6$ D_{4d}, of $B = 7$ dodecahedral symmetry, and of $B = 8$ D_{6d} symmetry [5,2], etc.

The minimisation, with the help of a 3-dimensional variational program [6], lowers the energies of these configurations by a few hundreds of MeV and shows that they become local minima in the $SU(3)$ configuration space. The knowledge of the so-called "flavour" moment of inertia and the Σ term allows then to estimate the flavour excitation energies. The mass splittings of the lowest states with different values of strangeness, charm or bottom are calculated within the rigid oscillator version of the bound state approach. The binding energies of baryonic systems (BS) with different values of flavours are also estimated.

To reduce theoretical uncertainties we consider the differences between the binding energies of BS with flavour F and the ground state for each value of B. These ground states are the deuteron for $B = 2$, the isodoublet ^{3}H–^{3}He for $B = 3$, ^{4}He for $B = 4$, etc. These differences, being free of many uncertainties and in particular of the poorly known loop corrections to classical masses, show the tendency of the flavoured BS to be more bound than the (u, d) ground states (for heavy flavours), or to be less bound, as for strangeness. Of course, we have made the assumption here that the ground states of multiskyrmions correspond to ordinary nuclei. However, this is a natural assumption if we believe that effective field theories describe nature.

In the next section we present a description of the static properties of multiskyrmions. Flavour excitation energies and zero mode corrections to the energies of multibaryons are considered in Sect. 3. Section 4 contains estimates of the binding energies of baryonic systems with different values of flavours, and our conclusions are given in Sect. 5.

2 Static properties of multiskyrmions

We consider here simple $SU(3)$ extensions of the Skyrme model [1]: we start with $SU(2)$ skyrmions (with flavours corresponding to (u,d) quarks) and extend them to various $SU(3)$ groups, such as (u,d,s), (u,d,c), or (u,d,b).

For the Lagrangian we take the usual expression of the Skyrme model which, in its well-known form, depends on parameters F_π, F_D and e and can be written in the following way [7]:

$$\begin{aligned}\mathcal{L} = &-\frac{F_\pi^2}{16}\mathrm{Tr} l_\mu l^\mu + \frac{1}{32e^2}\mathrm{Tr}[l_\mu, l_\nu]^2 \\ &+\frac{F_\pi^2 m_\pi^2}{16}\mathrm{Tr}(U+U^\dagger-2) \\ &+\frac{F_D^2 m_D^2 - F_\pi^2 m_\pi^2}{24}\mathrm{Tr}(1-\sqrt{3}\lambda_8)(U+U^\dagger-2) \\ &+\frac{F_D^2-F_\pi^2}{48}\mathrm{Tr}(1-\sqrt{3}\lambda_8)(Ul_\mu l^\mu + l_\mu l^\mu U^\dagger). \qquad (1)\end{aligned}$$

Here $U \in SU(3)$ is a unitary matrix incorporating chiral (meson) fields, and $l_\mu = U^\dagger \partial_\mu U$. In this model F_π is fixed at the physical value: $F_\pi = 186$ MeV and M_D is the mass of K, D or B meson. The ratios F_D/F_π are known to be 1.22 and 1.7 ± 0.2 for, respectively, kaons and D mesons.

The flavour symmetry breaking term (FSB) in the Lagrangian is of the usual form and is sufficient to describe the mass splittings of the octet and decuplets of baryons within the collective coordinate quantization approach [7].

The Wess–Zumino term, to be added to the action, which, as is well known [8], can be written as a 5-dimensional differential form, plays an important role in the quantization procedure:

$$S^{\mathrm{WZ}} = \frac{-\mathrm{i}N_c}{240\pi^2}\int_\Omega \mathrm{d}^5 x \epsilon^{\mu\nu\lambda\rho\sigma}\mathrm{Tr}(l_\mu l_\nu l_\lambda l_\rho l_\sigma), \qquad (2)$$

where Ω is a 5-dimensional region with the 4-dimensional space-time as its boundary and where l_μ are 5-dimensional extensions of $l_\mu = U^\dagger\partial_\mu U$. Action (2) is responsible for important topological properties of skyrmions, but it does not contribute to the static masses of classical configurations [8,9]. The variation of this action can be presented as a well-defined contribution to the Lagrangian (integral over the 4-dimensional space-time).

We begin our calculations, however, with $U \in SU(2)$. The classical mass of $SU(2)$ solitons, in the most general case, depends on three profile functions: f, α and β and is given by

$$\begin{aligned}M_{\mathrm{cl}} = \int \Big\{ &\frac{F_\pi^2}{8}[\vec{l}_1^2 + \vec{l}_2^2 + \vec{l}_3^2] \\ &+ \frac{1}{2e^2}[[\vec{l}_1\vec{l}_2]^2 + [\vec{l}_2\vec{l}_3]^2 + [\vec{l}_3\vec{l}_1]^2] \\ &+ \frac{1}{4}F_\pi^2 m_\pi^2 (1-c_f)\Big\}\mathrm{d}^3 r. \qquad (3)\end{aligned}$$

Here $\vec{l}_k$ are the $SU(2)$ chiral derivatives defined by $U^\dagger\vec{\partial}U = \mathrm{i}\vec{l}_k\tau_k$, where $k = 1,2,3$. The general parametrisation of U_0 for an $SU(2)$ soliton that we use here is given by $U_0 = c_f + s_f\vec{\tau}\vec{n}$ with $n_z = c_\alpha$, $n_x = s_\alpha c_\beta$, $n_y = s_\alpha s_\beta$, $s_f = \sin f$, $c_f = \cos f$, etc. For the rational map ansatz that we will use here as our starting configurations,

$$n_x = \frac{2\mathrm{Re}R(\xi)}{1+|R(\xi)|^2}, \quad n_y = \frac{2\mathrm{Im}R(\xi)}{1+|R(\xi)|^2},$$
$$n_z = \frac{1-|R(\xi)|^2}{1+|R(\xi)|^2},$$

where $R(\xi)$ is a ratio of polynomials (of the maximal degree B) in the variable $\xi = \mathrm{tg}(\theta/2)\exp(\mathrm{i}\phi)$. θ and ϕ are the polar and azimuthal angles defining the direction of the radius vector $\vec{r}$. The explicit form of $R(\xi)$ is given in [2] for different values of B.

The "flavour" moment of inertia plays a very important role in the procedure of $SU(3)$ quantization [10–18]; see (9) and (10) below, and for arbitrary $SU(2)$ skyrmions is given by [17,19]

$$\begin{aligned}\Theta_F = \frac{1}{8}\int(1-c_f)\Big[F_D^2 + \frac{1}{e^2}\Big((\vec{\partial}f)^2 + s_f^2(\vec{\partial}\alpha)^2 \\ + s_f^2 s_\alpha^2(\vec{\partial}\beta)^2\Big)\Big]\mathrm{d}^3\vec{r}. \qquad (4a)\end{aligned}$$

It is simply connected with $\Theta_F^{(0)}$ of the flavour symmetric case ($F_D = F_\pi$):

$$\Theta_F = \Theta_F^{(0)} + (F_D^2/F_\pi^2 - 1)\Gamma/4, \qquad (4b)$$

with Γ defined in (5) below. The isotopic moments of inertia are the components of the corresponding tensor of inertia. They have been discussed in many papers, see e.g. [9–12], so we will not present them here. For the majority of multiskyrmions we are discussing here, this tensor of inertia is close to the unit matrix multiplied by the isotopic moment of inertia Θ_T. This is exactly the case for $B=1$ and, to within a good accuracy, for $B = 3,7$. Considerable deviations take place for the $B=2$ torus, and smaller ones for $B = 4,5,6$ and 8; see Table 1. The quantity Γ (or the Σ term), which defines the contribution of the mass term to the classical mass of solitons, and $\tilde{\Gamma}$ are used directly in the quantization procedure. They are given by

$$\Gamma = \frac{F_\pi^2}{2}\int(1-c_f)\mathrm{d}^3\vec{r}, \qquad (5)$$
$$\tilde{\Gamma} = \frac{1}{4}\int c_f[(\vec{\partial}f)^2 + s_f^2(\vec{\partial}\alpha)^2 + s_f^2 s_\alpha^2(\vec{\partial}\beta)^2]\mathrm{d}^3\vec{r}.$$

The following relation can also be established: $\tilde{\Gamma} = 2(M_{\mathrm{cl}}^{(2)}/F_\pi^2 - e^2\Theta_F^{\mathrm{Sk}})$, where $M_{\mathrm{cl}}^{(2)}$ is the second-order contribution to the classical mass of the soliton, and Θ_F^{Sk} is

Table 1. Characteristics of the bound states of skyrmions with baryon numbers up to $B = 8$. The classical mass of solitons $M_{\rm cl}$ is in GeV, moments of inertia Θ_F, Θ_T and $\Theta_{T,3}$, Γ and $\tilde\Gamma$ - in GeV^{-1}, the excitation frequencies for flavour F, ω_F in GeV. $c_{s,c,b}$ and $\bar c_{s,c}$ are the hyperfine splitting constants for multibaryons defined in (21a) and (21b). The constant $\bar c_b$ is close to 0.99 for all B and is not included in the Table. The external parameters of the model are $F_\pi = 186\, MeV, e = 4.12$. The accuracy of calculations is better than 1% for the masses and few % for other quantities. The $B = 1$ quantities as well as $B = 2$ quantities for the torus calculated previously, are shown for comparison

B	$M_{\rm cl}$	$\Theta_F^{(0)}$	Θ_T	$\Theta_{T,3}$	Γ	$\tilde\Gamma$	ω_s	ω_c	ω_b	c_s	c_c	c_b	$\bar c_s$	$\bar c_c$
1	1.702	2.04	5.55	5.55	4.83	15.6	0.309	1.542	4.82	0.28	0.27	0.52	0.54	0.91
2	3.26	4.18	11.5	7.38	9.35	22	0.293	1.511	4.76	0.27	0.24	0.49	0.53	0.90
3	4.80	6.34	14.4	14.4	14.0	27	0.289	1.504	4.75	0.40	0.37	0.58	0.60	0.92
4	6.20	8.27	16.8	20.3	18.0	31	0.283	1.493	4.74	0.47	0.44	0.62	0.64	0.92
5	7.78	10.8	23.5	19.5	23.8	35	0.287	1.505	4.75	0.42	0.40	0.60	0.62	0.92
6	9.24	13.1	25.4	27.7	29.0	38	0.287	1.504	4.75	0.48	0.46	0.63	0.67	0.93
7	10.6	14.7	28.9	28.9	32.3	44	0.282	1.497	4.75	0.48	0.46	0.64	0.66	0.93
8	12.2	17.4	33.4	31.4	38.9	47	0.288	1.510	4.79	0.49	0.47	0.64	0.67	0.93

the Skyrme term contribution to the flavour moment of inertia. The calculated masses of solitons, moments of inertia Θ_F, Θ_T, Γ or Σ term and $\tilde\Gamma$ are presented in Table 1 above.

As can be seen from Tables 1 and 2, there are two "islands" of stability for the baryon numbers considered here: at $B = 4$, which is not unexpected, and for $B = 7$, and this appears to be new. So far, this property seems to be specific to the Skyrme model. The difference between Θ_T and $\Theta_{T,3}$ is maximal for the toroidal $B = 2$ configuration and decreases with increasing B. It vanishes for $B = 3$ and 7. The accuracy of the calculation decreases with increasing B. It is difficult to estimate this accuracy for such quantities as ω_F and c_F (see Sect. 3) as their values depend also on the particular method of calculation – the rigid oscillator model in our case.

The behaviour of static properties of multiskyrmions and flavour excitation frequencies shown in Table 1 is similar to that obtained in [22] for toroidal configurations with $B = 2, 3, 4$. We note that the flavour moment of inertia $\Theta_{F,B}$ and the sigma term Γ_B increase with B almost proportionally to B.

3 Flavour excitation frequencies and $\sim 1/N_c$ zero mode corrections

To quantize the solitons in their $SU(3)$ configuration space, in the spirit of the bound state approach to the description of strangeness proposed in [13,14] and used in [15–17], we consider the collective coordinate motion of the meson fields incorporated into the matrix U:

$$U(r,t) = R(t)U_0(O(t)\vec r)R^\dagger(t), \quad R(t) = A(t)S(t), \quad (6)$$

where U_0 is the $SU(2)$ soliton embedded into $SU(3)$ in the usual way (into the upper left hand corner) and $A(t) \in SU(2)$ describes $SU(2)$ rotations. Moreover, $S(t) \in SU(3)$ describes rotations in the "strange", "charm" or "bottom" directions and $O(t)$ describes rigid rotations in real space. For definiteness we consider the extension of the (u, d) $SU(2)$ Skyrme model in the (u, d, s) direction, when D is the field of K mesons, but it is clear that quite similar extensions can also be made in the directions of charm or bottom. So

$$S(t) = \exp(\mathrm{i}\mathcal{D}(t)), \quad \mathcal{D}(t) = \sum_{a=4,...7} D_a(t)\lambda_a, \quad (7)$$

where λ_a are the Gell-Mann matrices of the (u, d, s), (u, d, c) or (u, d, b) $SU(3)$ groups. The (u, d, c) and (u, d, b) $SU(3)$ groups are quite analogous to the (u, d, s) one. For the (u, d, c) group a simple redefiniton of hypercharge should be made. For the (u, d, s) group, $D_4 = (K^+ + K^-)/2^{1/2}$, $D_5 = \mathrm{i}(K^+ - K^-)/2^{1/2}$, etc. For the (u, d, c) group $D_4 = (D^0 + \bar D^0)/2^{1/2}$, etc.

The angular velocities of the isospin rotations $\vec\omega$ are defined in the standard way [9]: $A^\dagger \dot A = -\mathrm{i}\vec\omega\vec\tau/2$. We shall not consider here, in much detail, the usual space rotations because the corresponding moments of inertia for BS are much greater than the isospin moments of inertia, and for the lowest possible values of the angular momentum J, the corresponding quantum correction is either exactly zero (for even B), or small; see also (17a), (17b), (21a) and (21b) below.

The field D is small in magnitude. In fact, it is at least of order $1/N_c^{1/2}$, where N_c is the number of colours in QCD; see (14). Therefore, the expansion of the matrix S in the powers of D can be made safely.

The mass term of the Lagrangian (1) can be calculated exactly, without expansion in the powers of the field D, because the matrix S is given by $S = 1 - \mathrm{i}\mathcal{D}\sin d/d - \mathcal{D}^2(1 - \cos d)/d^2$ with $d^2 = \mathrm{Tr}\mathcal{D}^2$. We find that

$$\Delta\mathcal{L}_M = -\frac{F_D^2 m_D^2 - F_\pi^2 m_\pi^2}{4}(1 - c_f)s_d^2. \quad (8)$$

The expansion of this term can be done easily up to any order in d. The comparison of this expression with ΔL_M, within the collective coordinate approach of the quantization of $SU(2)$ solitons in the $SU(3)$ configuration space [10–12], allows us to establish the relation $\sin^2 d = \sin^2 \nu$, where ν is the angle of the λ_4 rotation, or the rotation into the "strange" ("charm", "bottom") direction.

After some calculations we find that the Lagrangian of the model, to the lowest order in the field D, can be written as

$$L = -M_{\mathrm{cl},B} + 4\Theta_{F,B}\dot{D}^\dagger\dot{D} - \left[\Gamma_B\left(\frac{F_D^2}{F_\pi^2}m_D^2 - m_\pi^2\right) + \tilde{\Gamma}_B(F_D^2 - F_\pi^2)\right] D^\dagger D - \mathrm{i}\frac{N_cB}{2}(D^\dagger\dot{D} - \dot{D}^\dagger D). \tag{9}$$

Here and below D is the doublet $(K^+, K^0)^T$, $(D^0, D^-)^T$, or $(B^+, B^0)^T$; $D^\dagger D = d^2/2$. We have kept the standard notation for the moment of inertia of the rotation into the "flavour" direction Θ_F for Θ_c, Θ_b or Θ_s [10–12]; different notations are used in [15,16] (the index c denotes the charm quantum number, except in N_c). The contribution proportional to $\tilde{\Gamma}_B$ is suppressed in comparison with the term $\sim \Gamma$ by a small factor $\sim (F_D^2 - F_\pi^2)/m_D^2$, and is more important for the strangeness.

The term proportional to N_cB in (9) arises from the Wess–Zumino term in the action and is responsible for the difference of the excitation energies of strangeness and antistrangeness (flavour and antiflavour in the general case) [13–16].

Following the canonical quantization procedure the Hamiltonian of the system, including the terms of the order of N_c^0, takes the form [15,16]

$$H_B = M_{\mathrm{cl},B} + \frac{1}{4\Theta_{F,B}}\Pi^\dagger\Pi + \left[\Gamma_B\bar{m}_D^2 + \tilde{\Gamma}_B(F_D^2 - F_\pi^2) + \frac{N_c^2B^2}{16\Theta_{F,B}}\right] D^\dagger D + \mathrm{i}\frac{N_cB}{8\Theta_{F,B}}(D^\dagger\Pi - \Pi^\dagger D), \tag{10}$$

where $\bar{m}_D^2 = (F_D^2/F_\pi^2)m_D^2 - m_\pi^2$. The momentum Π is canonically conjugate to D (see (18) below). Equation (10) describes an oscillator-type motion of the field D in the background formed by the (u,d) $SU(2)$ soliton. After the diagonalization, which can be done explicitly following [15, 16], the normal-ordered Hamiltonian can be written as

$$H_B = M_{\mathrm{cl},B} + \omega_{F,B}a^\dagger a + \bar{\omega}_{F,B}b^\dagger b + O(1/N_c), \tag{11}$$

with $a^\dagger$, $b^\dagger$ being the operators of creation of strangeness (i.e. of antikaons) and antistrangeness, (flavour and antiflavour) quantum number, $\omega_{F,B}$ and $\bar{\omega}_{F,B}$ being the frequencies of flavour (antiflavour) excitations. D and Π are connected with a and b in the following way [15,16]:

$$D^i = \frac{1}{\sqrt{N_cB\mu_{F,B}}}(b^i + a^{\dagger i}), \quad \Pi^i = \frac{\sqrt{N_cB\mu_{F,B}}}{2\mathrm{i}}(b^i - a^{\dagger i}) \tag{12}$$

with

$$\mu_{F,B} = [1 + 16(\bar{m}_D^2\Gamma_B + (F_D^2 - F_\pi^2)\tilde{\Gamma}_B)\Theta_{F,B}/(N_cB)^2]^{1/2}. \tag{13}$$

For the lowest states the values of D are small:

$$D \sim [16\Gamma_B\Theta_{F,B}\bar{m}_D^2 + N_c^2B^2]^{-1/4}, \tag{14}$$

and increase, with increasing flavour number $|F|$, like $(2|F|+1)^{1/2}$. As was noted in [16], deviations of the field D from the vacuum decrease with increasing mass m_D, as well as with increasing number of colours N_c, and the method works well for any m_D (and also for the charm and bottom quantum numbers).

The excitation frequencies ω and $\bar{\omega}$ are

$$\omega_{F,B} = \frac{N_cB}{8\Theta_{F,B}}(\mu_{F,B} - 1), \quad \bar{\omega}_{F,B} = \frac{N_cB}{8\Theta_{F,B}}(\mu_{F,B} + 1). \tag{15}$$

As was observed in [17], the difference $\bar{\omega}_{F,B} - \omega_{F,B} = N_cB/(4\Theta_{F,B})$ coincides, to the leading order in N_c, with the expression obtained in the collective coordinate approach [18,19]. At large m_D, we have $\mu_{F,B} \simeq 4\bar{m}_D(\Gamma_B\Theta_{F,B})^{1/2}/(N_cB)$, and for the flavour excitation energy we obtain ($N_c = 3$)

$$\omega_{F,B} \simeq \frac{\bar{m}_D}{2}\left(\frac{\Gamma_B}{\Theta_{F,B}}\right)^{1/2} - \frac{3}{8}\frac{B}{\Theta_{F,B}}. \tag{16a}$$

Since $\bar{m}_D < F_Dm_D/F_\pi$ and $\Theta_{F,B} > F_D^2\Gamma_B/(4F_\pi^2)$, see (4a) and (4b), it follows from (16a) that the excitation energies $\omega_{F,B}$ are always smaller than the corresponding meson masses m_D, i.e. we have rigorously established, within the model used here, the binding of flavoured mesons by an $SU(2)$ skyrmion.

For the difference $\omega_{F,1} - \omega_{F,B}$ we obtain

$$\omega_{F,1} - \omega_{F,B} \simeq \frac{\bar{m}_D}{2}\left[\left(\frac{\Gamma_1}{\Theta_{F,1}}\right)^{1/2} - \left(\frac{\Gamma_B}{\Theta_{F,B}}\right)^{1/2}\right] + \frac{3}{8}\left(\frac{B}{\Theta_{F,B}} - \frac{1}{\Theta_{F,1}}\right). \tag{16b}$$

Obviously, at large m_D, the first term in (16b) dominates and is positive if $\Gamma_1/\Theta_{F,1} \geq \Gamma_B/\Theta_{F,B}$. This is confirmed by looking at Table 1. Note also that the bracket in the first term in (16b) does not depend on the parameters of the model if the background $SU(2)$ soliton is calculated in the chirally symmetrical limit as both Γ and Θ scale like $\sim 1/(F_\pi e^3)$. In a realistic case when the physical pion mass is included in (3) there is some weak dependence on the parameters of the model.

The FSB in the flavour decay constants, i.e. the fact that $F_K/F_\pi \simeq 1.22$ and $F_D/F_\pi = 1.7 \pm 0.2$ should also be taken into account. In the Skyrme model this fact leads to the increase of the flavour excitation frequencies which changes the spectra of flavoured (c, b) baryons and puts them in a better agreement with the data [20,21]. It also leads to some changes of the total binding energies of BS [17]. This is partly due to the large contribution of the Skyrme term to the flavour moment of inertia Θ_F. Note that in [16] the FSB in the strangeness decay constant was not taken into account, and this has led to the underestimation of the strangeness excitation energies. Heavy flavours (c, b) have not been considered in these papers.

The terms of the order of N_c^{-1} in the Hamiltonian, which depend on the angular velocities of rotations in the isospin and the usual space and which describe the zero mode contributions, are not crucial but are important for the numerical estimates of the spectra of baryonic systems. To calculate them one should first obtain the Lagrangian of BS including all the terms up to $O(1/N_c)$. The Lagrangian can be written in a compact form as

$$\begin{aligned} L \simeq & -M_{\mathrm{cl}} + 4\Theta_{F,B}\left[\dot{D}^\dagger\dot{D}\left(1-\frac{d^2}{3}\right)\right. \\ & \left. -\frac{2}{3}(D^\dagger\dot{D}\dot{D}^\dagger D - (D^\dagger\dot{D})^2 - (\dot{D}^\dagger D)^2)\right] \\ & + 2\Theta_{F,B}(\vec{\omega}\vec{\beta}) + \frac{\Theta_{T,B}}{2}(\vec{\omega}-\vec{\beta})^2 \\ & - [\Gamma_B\tilde{m}_D^2 + (F_D^2 - F_\pi^2)\tilde{\Gamma}_B]D^\dagger D\left(1-\frac{d^2}{3}\right) \\ & + \mathrm{i}\frac{N_cB}{3}\left(1-\frac{d^2}{3}\right)(\dot{D}^\dagger D - D^\dagger\dot{D}) - \frac{N_cB}{2}\vec{\omega}D^\dagger\vec{\tau}D, \end{aligned} \tag{17a}$$

where $d^2 = 2D^\dagger D$ and

$$\vec{\beta} = -\mathrm{i}(\dot{D}^\dagger\vec{\tau}D - D^\dagger\vec{\tau}\dot{D}).$$

For the axially symmetrical configurations, like the $B = 2$ torus, the term $\Theta_{T,B}(\omega_3-\beta_3)^2/2$ in (17a) should be replaced by

$$\delta L = \frac{\Theta_{3,B}}{2}(\omega_3 - n\Omega_3 - \beta_3)^2 + \frac{\Theta_{J,B}}{2}(\Omega_1^2+\Omega_2^2), \tag{17b}$$

where Ω_i are the components of the angular velocities of rotation in the usual space, $\dot{O}_{in}O_{kn} = \epsilon_{ikl}\Omega_l$. Taking into account the terms $\sim 1/N_c$ we find that the canonical variable Π conjugate to D is

$$\begin{aligned} \Pi = & \frac{\partial L}{\partial \dot{D}^\dagger} \\ = & 4\Theta_{F,B}\left[\dot{D}\left(1-\frac{\mathrm{d}^2}{3}\right) - \frac{2}{3}D^\dagger\dot{D}D + \frac{4}{3}\dot{D}^\dagger DD\right] \\ & + \mathrm{i}(\Theta_{T,B} - 2\Theta_{F,B})\vec{\omega}\vec{\tau}D - \mathrm{i}\Theta_{T,B}\vec{\beta}\vec{\tau}D \\ & + \mathrm{i}\frac{N_cB}{2}\left(1-\frac{d^2}{3}\right)D. \end{aligned} \tag{18}$$

From (17a) the body-fixed isospin operator is

$$\begin{aligned} \vec{I}^{\mathrm{bf}} = & \partial L/\partial\vec{\omega} = \Theta_{T,B}\vec{\omega} + (2\Theta_{F,B} - \Theta_{T,B})\vec{\beta} \\ & - \frac{N_cB}{2}D^\dagger\vec{\tau}D. \end{aligned} \tag{19}$$

Using the identities

$$-\mathrm{i}\vec{\beta}\vec{\tau}D = 2D^\dagger D\dot{D} - (\dot{D}^\dagger D + D^\dagger\dot{D})D \tag{20a}$$

and

$$\vec{\beta}^2 = 4D^\dagger D\dot{D}^\dagger\dot{D} - (\dot{D}^\dagger D + D^\dagger\dot{D})^2, \tag{20b}$$

we find that the $\sim 1/N_c$ zero mode quantum corrections to the energies of skyrmions can be estimated [15,16] as

$$\begin{aligned} \Delta E_{1/N_c} = & \frac{1}{2\Theta_{T,B}}[c_{F,B}T_{\mathrm{r}}(T_{\mathrm{r}}+1) + (1-c_{F,B})I(I+1) \\ & + (\bar{c}_{F,B} - c_{F,B})I_F(I_F+1)], \end{aligned} \tag{21a}$$

where $I = I^{\mathrm{bf}}$ is the value of the isospin of the baryon or BS, which can be written also as

$$\vec{I}^{\mathrm{bf}} = \Theta_T\vec{\omega} + \left(1-\frac{\Theta_T}{2\Theta_F}\right)\vec{I}_F - \frac{N_cB\Theta_T}{4\Theta_F}D^\dagger\vec{\tau}D, \tag{22}$$

with the operator $\hat{\vec{I}}_F = (b^\dagger\vec{\tau}b - a^T\vec{\tau}a^{\dagger T})/2$.

T_{r} is the quantity analogous to the "right" isospin T_{r}, in the collective coordinate approach [10,18], and $\vec{T}_{\mathrm{r}} = \vec{I}^{\mathrm{bf}} - \vec{I}_F$. The hyperfine structure constants $c_{F,B}$ and $\bar{c}_{F,B}$ are defined by the relations

$$\begin{aligned} 1 - c_{F,B} = & \frac{\Theta_{T,B}}{2\Theta_{F,B}\mu_{F,B}}(\mu_{F,B}-1), \\ 1 - \bar{c}_{F,B} = & \frac{\Theta_{T,B}}{\Theta_{F,B}(\mu_{F,B})^2}(\mu_{F,B}-1). \end{aligned} \tag{23}$$

To take into account the usual space rotations the J-dependent terms should be added to (21a). For the axially symmetrical configurations, like the $B = 2$ torus, they are equal to [18,16]

$$\begin{aligned} \Delta E^J_{1/N_c} = & \left(\frac{1}{2n^2\Theta_{3,B}} - \frac{1}{2n^2\Theta_{T,B}} - \frac{1}{2\Theta_{J,B}}\right)(J_3^{\mathrm{bf}})^2 \\ & + \frac{J(J+1)}{2\Theta_{J,B}}, \end{aligned} \tag{21b}$$

with $\Theta_{J,B}$ being the moment of inertia corresponding to the usual space rotations – the orbital moment of inertia, which is known to increase with increasing B number almost proportionally to B^2 [17,23]. For such configurations the body-fixed 3–d component of the angular momentum J_3^{bf} and the nonstrange part of the 3–d component of the isospin (also body-fixed) are connected by the relation $J_3^{\mathrm{bf}} = -nT_{\mathrm{r},3}^{\mathrm{bf}}$ (see e.g. [18,16] and references therein). Realistic cases of multiskyrmions are intermediate between the case of incoherence of usual space and isospace rotations and the complete coherence, as in (21b) for the rotation relative to the axis of axial symmetry. However, the J-dependent terms of the type (21b) cancel in the differences of energies of states which belong to the same $SU(3)$ multiplet, i.e. which have the same values of $J, (p, q)$ and $T_{\mathrm{r},3}$.

In the case of antiflavour excitations we obtain the same formulas with the substitution $\omega \to \bar{\omega}$ in the expression for the energy of the state and $\mu \to -\mu$ in (23). For example,

$$\bar{c}_{\bar{F},B} = 1 + \frac{\Theta_{T,B}}{\Theta_{F,B}\mu_{F,B}^2}(\mu_{F,B} + 1). \tag{24}$$

The excitation energies for antiflavours are close to $\sim 0.59\,\mathrm{GeV}$ for antistrangeness, $\sim 1.75\,\mathrm{GeV}$ for anticharm and to $\sim 4.95\,\mathrm{GeV}$ for antibottom. However, these numbers should be considered as lower bounds only since to calculate them we have used a simplified version of the bound state soliton model.

4 Estimates of the spectra of multibaryons with strangeness, charm or bottom

In the bound state soliton model, and in its rigid oscillator version as well, the states predicted do not correspond to the definite $SU(3)$ or $SU(4)$ representations. How this can be remedied was shown in [16]; see also (26)–(29) below. The quantization condition $(p + 2q)/3 = B$ [10], for arbitrary N_c, changes to $(p + 2q) = N_cB + 3n_{q\bar{q}}$, where $n_{q\bar{q}}$ is the number of additional quark–antiquark pairs present in the quantized state [18]. For example, the state with $B = 1$, $|F| = 1$, $I = 0$ and $n_{q\bar{q}} = 0$ should belong to the octet of (u, d, s), or (u, d, c), $SU(3)$ group, if $N_c = 3$; see also [16]. The state with $B = 2$, $|F| = 2$ and $I = 0$ should belong to the 27-plet of the corresponding group, etc. The states having antiflavour quantum number, i.e. positive strangeness or bottom quantum number or negative charm should have $n_{q\bar{q}} \geq |F|$ [18]. If $\Theta_F \to \infty$, (21a) and (21b) go over into the expressions obtained within the collective coordinate approach [10,17]. In a realistic case, with $\Theta_T/\Theta_F^{(0)} \sim 2$–$2.7$, the structure of (21a) and (21b) is more complicated.

First we consider quantized states of BS which belong to the lowest possible $SU(3)$ irreps (p, q) for each value of the baryon number, $p + 2q = 3B$: $p = 0, q = 3B/2$ for even B, and $p = 1, q = (3B-1)/2$ for odd B. For $B = 3, 5$ and 7 they are $\overline{35}, \overline{80}$ and $\overline{143}$-plets, for $B = 2, 4, 6$ and 8–$\overline{10}$, $\overline{28}$, $\overline{55}$ and $\overline{91}$-plets. Since we are interested in the lowest energy states, we discuss here the baryonic systems with the lowest allowed angular momentum, i.e. $J = 0$, for $B = 2$, $4, 6$ and 8. For odd B the quantization of BS encounters some difficulties (see [23]), but the correction to the energy of quantized states due to the nonzero angular momentum is small and decreases with increasing B since the corresponding moment of inertia increases proportionally to $\sim B^2$ [22,23]. Moreover, the J-dependent correction to the energy cancels in the differences of energies of flavoured and flavourless states.

For the energy difference between the state with flavour F belonging to the (p, q) irrep, and the ground state with $F = 0$ and the same angular momentum and (p, q) we obtain

$$\Delta E_{B,F} = |F|\omega_{F,B} + \frac{\mu_{F,B} - 1}{4\mu_{F,B}\Theta_{F,B}}[I(I+1) - T_\mathrm{r}(T_\mathrm{r}+1)] + \frac{(\mu_{F,B} - 1)(\mu_{F,B} - 2)}{4\mu_{F,B}^2\Theta_{F,B}} I_F(I_F + 1), \tag{25}$$

where $T_\mathrm{r} = p/2$ and usually $I_F = I - T_\mathrm{r}$. According to (25), the mass splittings within the same $SU(3)$ irrep (p, q) are defined by flavour inertia $\Theta_{F,B}$ and also Γ_B which enters through $\mu_{F,B}$. The moment of inertia Θ_T enters the difference of energies between different irreps. Obviously, for “minimal” BS, i.e. those which do not contain additional quark–antiquark pairs,

$$T_\mathrm{r} \leq 3B/2. \tag{26}$$

The isospin carried by $|F|$ flavoured mesons bound by (u, d) solitons satisfies another obvious relation:

$$I_F \leq |F|/2. \tag{27}$$

Simple arguments allow us also to get the following restrictions on the total isospin of BS:

$$|T_\mathrm{r} - |F|/2| \leq I \leq T_\mathrm{r} + |F|/2 \tag{28}$$

and

$$I \leq (3B - |F|)/2. \tag{29}$$

The lowest of the two upper bounds should be taken as the final upper bound. It is easy to check that our bounds correspond to the known $SU(3)$ multiplets for each value of T_r.

For fixed B, $SU(3)$ multiplet (p, q) and $I_F = |F|/2$, the state of the minimal energy is the state of the lowest isospin I, as follows from (25). But to reach the conclusion concerning the stability of any state against its decay due to strong interactions one should compare its energy with the energy of possible final states consisting of separate baryons and satisfying the requirement of conservation of charge, flavour and isospin. Since different baryons have different energies, the most stable state will not necessarily be the multibaryon of the lowest energy.

For the $B = 1$ case, the difference of masses within the octet of baryons, Λ_F and N, Σ_F and Λ_F, is

$$\Delta M_{\Lambda_F N} = \omega_{F,1} - \frac{3(1 - \bar{c}_{F,1})}{8\Theta_{T,1}} = \omega_{F,1} - \frac{3(\mu_{F,1} - 1)}{8\mu_{F,1}^2\Theta_{F,1}},$$

$$\Delta M_{\Sigma_F\Lambda_F} = \frac{(1 - c_{F,1})}{\Theta_{T,1}} = \frac{\mu_{F,1} - 1}{2\mu_{F,1}\Theta_{F,1}},$$

$$M_\Xi + M_N - 2M_\Lambda = \frac{3(\mu_{F,1} - 1)}{4\mu_{F,1}^2\Theta_{F,1}}. \tag{30}$$

Clearly, the binding energy of the multiskyrmion cancels out in (25). For states with maximal isospin $I = T_\mathrm{r}+|F|/2$, the energy difference (25) can be simplified to

$$\Delta E_{B,F} = |F| \left[\omega_{F,B} + T_\mathrm{r} \frac{\mu_{F,B}-1}{4\mu_{F,B}\Theta_{F,B}} + \frac{(|F|+2)}{8\Theta_{F,B}} \frac{(\mu_{F,B}-1)^2}{\mu_{F,B}^2} \right]. \tag{31}$$

For even B, $T_\mathrm{r} = 0$; for odd B, we should take $T_\mathrm{r} = 1/2$ for the lowest $SU(3)$ irreps.

It follows from (30) and (31) that when some nucleons are replaced by flavoured hyperons (Λs) in BS the binding energy of the system changes by

$$\Delta\epsilon_{B,F} = |F| \left[\omega_{F,1} - \omega_{F,B} - \frac{3(\mu_{F,1}-1)}{8\mu_{F,1}^2\Theta_{F,1}} - T_\mathrm{r} \frac{\mu_{F,B}-1}{4\mu_{F,B}\Theta_{F,B}} - \frac{(|F|+2)}{8\Theta_{F,B}} \frac{(\mu_{F,B}-1)^2}{\mu_{F,B}^2} \right]. \tag{32}$$

This expression is valid for $|F| = 1$ or when the number of nucleons is large enough to satisfy the isospin conservation requirement. In some remaining cases it should be modified. For $|F| = 2$, in view of the isospin conservation law we should consider $\Sigma_F \Lambda_F$ for $B = 2$ ($I = 1$) and $\Xi_F NN$ for $B = 3$ ($I = 3/2$) as the available final states. Similarly, for $|F| = 3$ and 4 and small B numbers we should insert $\Sigma_F \Lambda_F$ or $\Xi_F N$ pairs instead of $\Lambda_F \Lambda_F$ to satisfy the isospin conservation. As a result, the binding energy relative to the transition to the mentioned states increases in comparison with only the $\Lambda_F N$ final states. A transition into such states will be possible, however, due to the electromagnetic interaction which does not conserve the isospin. For example, the state with $B = 3$, $S = -3$ is bound relative to the $\Xi\Sigma N$ final state by 23 MeV, but could decay electromagnetically into the $\Lambda\Lambda\Sigma$ final state if it is allowed by the electric charge conservation law. The state with $B = 3$, $S = -4$, $I = 5/2$ is stable with respect to the strong decay into the $\Xi\Sigma\Sigma$ state, but could decay electromagnetically into $\Xi\Sigma\Lambda$.

For strangeness the binding energy difference (32) is mostly negative indicating that stranglets should have binding energies smaller than those of nuclei, or could be unbound. Since $\Theta_{F,B}$ and $\Theta_{T,B}$ increase with increasing B and m_D, this leads to the increase of binding with increasing B and with the mass of the flavoured state, in agreement with [17]. For charm and bottom (32) is positive for $B \geq 3$ or 4. It follows from Table 2 that dibaryons with strangeness or charm are probably unbound, but those with $b = -1$ or $b = -2$ could be bound. The multibaryons with $B \geq 4$ and $S = -1$ can be bound, as well as multibaryons with $c = 1, 2$ or 3, or bottom $b = -1, -2$.

Had the moments of inertia of BS at small values of B been proportional to the baryon number B, then the values of μ, excitation frequencies ω_F and coefficients c would not have depended on B at all. In this case the binding energy would have consisted only of its classical part and a contribution from zero modes; the difference of ω's would have been absent in this case.

Nuclear fragments with sufficiently large values of the strangeness (or bottom) may be found in experiments as fragments with negative charge Q, according to the well-known relation, $Q = T_3 + (B + S)/2$ (similarly for the bottom number). Recently one event of a long lived nuclear fragment with mass about 7.4 GeV was reported in [25]. Using the above formulas it is not difficult to establish that this fragment may be the state with $B = -S = 6$, or $B = 7$ and strangeness $S = -4$, or -3; see also Table 3 below. Greater strangeness values are not excluded since the method used here overestimates the flavour excitation energies, especially for smaller baryon numbers and for the strangeness quantum number.

Another case of interest involves considering the BS with isospin $I = 0$. In this case $I_F = T_\mathrm{r} = |F|/2$, so such states do not belong to the lowest possible $SU(3)$ multiplet for each value of B (except for the case $|F| = 1$). For the energy difference between this state and a flavourless state belonging to the same $SU(3)$ irrep it is easy to obtain

$$\Delta E_{B,F} = |F| \left[\omega_{F,B} - \frac{(|F|+2)}{8\Theta_{F,B}} \frac{(\mu_{F,B}-1)}{\mu_{F,B}^2} \right]. \tag{33}$$

For the difference of binding energies of such a state and the ground (u, d) state with lowest values $(p^{\min}, q^{\min})$ we have the following estimate:

$$\Delta\epsilon_{B,F} = |F| \left[\omega_{F,1} - \omega_{F,B} - \frac{3(\mu_{F,1}-1)}{8\Theta_{F,1}\mu_{F,1}^2} + \frac{(|F|+2)}{8\Theta_{F,B}} \frac{(\mu_{F,B}-1)}{\mu_{F,B}^2} \right] - \frac{1}{2\Theta_{T,B}} [|F|(|F|+2)/4 - T_\mathrm{r}^{\min}(T_\mathrm{r}^{\min}+1)], \tag{34}$$

where $T_\mathrm{r}^{\min} = 0$, or $1/2$. Using this formula we find the values given in Table 3. For example, the $B = 2$, $|F| = 2$ state discussed previously in [18] and later in [16] belongs to the 27-plet of the corresponding $SU(3)$ group. In the case of strangeness it has already appeared, probably, as a virtual level in the $\Lambda\Lambda$ system [24].

We can see from Table 3 that for $|F| = 1$ and partly for $|F| = 2$ the isoscalar states have more chance to be bound than states presented in Table 2. The $B = 7, S = -3, I = 0$ state has a binding energy smaller than the (u, d) nucleus by 42 MeV, i.e. it can be stable with respect to the strong decay, if we take into account the uncertainty of our estimates (recall that the nucleus ^{7}Li has a total binding energy of 39 MeV.) The state with isospin equal to 2 – the maximal value for $S = -3$ within the $(1, 10)$ $SU(3)$ multiplet – has a somewhat greater energy; see Table 2. The difference of energies of states with isospin $I = I^{\max} = T_\mathrm{r} + |F|/2$ and $I = 0$, and the same value of F can be written as

$$E^{I^{\max}} - E^{I=0} = \Delta\epsilon_{B,F}^{I=0} - \Delta\epsilon_{B,F}^{I^{\max}}$$

Table 2. The binding energy differences $\Delta\epsilon_{s,c,b}$ are the changes of binding energies of lowest BS with flavour s, c or b and isospin $I = T_{\rm r} + |F|/2$ in comparison with the usual u, d nuclei, for the flavour numbers $S = -1, -2, -3$ and -4, $c = 1, 2$ and 3, $b = -1$ and -2 (see (32)). The $SU(3)$ multiplets are $(p, q) = (0, 3B/2)$ for even B and $(p, q) = (1, (3B-1)/2)$ for odd B

B	$\Delta\epsilon_{s=-1}$	$\Delta\epsilon_{c=1}$	$\Delta\epsilon_{b=-1}$	$\Delta\epsilon_{s=-2}$	$\Delta\epsilon_{c=2}$	$\Delta\epsilon_{b=-2}$	$\Delta\epsilon_{s=-3}$	$\Delta\epsilon_{c=3}$	$\Delta\epsilon_{s=-4}$
2	−0.047	−0.027	0.02	−0.053	−0.075	0.02	−0.013	−0.039	–
3	−0.042	−0.010	0.04	−0.036	−0.027	0.06	0.023	0.079	0.070
4	−0.020	0.019	0.06	−0.051	0.022	0.10	−0.030	0.026	−0.02
5	−0.027	0.006	0.05	−0.063	0.001	0.08	−0.046	0.031	−0.04
6	−0.019	0.016	0.05	−0.045	0.023	0.10	−0.078	0.028	−0.05
7	−0.016	0.021	0.06	−0.041	0.033	0.11	−0.070	0.037	−0.04
8	−0.017	0.014	0.02	−0.040	0.021	0.03	−0.068	0.020	−0.10

Table 3. The binding energies differences of lowest flavoured BS with isospin $I = 0$ and the ground state with the same value of B and $I = 0$ or $I = 1/2$; see (34). The first three columns are for $|F| = 1$, the next three columns for $|F| = 2$, and the next three for $|F| = 3$. The state with the value of flavour $|F|$ belongs to the $SU(3)$ multiplet with $T_{\rm r} = |F|/2$. In the last column the binding energies differences are shown for the isoscalar electrically neutral states with $S = -B$. For $|F| \geq 3$ all estimates are very approximate

B	$\Delta\epsilon_{s=-1}$	$\Delta\epsilon_{c=1}$	$\Delta\epsilon_{b=-1}$	$\Delta\epsilon_{s=-2}$	$\Delta\epsilon_{c=2}$	$\Delta\epsilon_{b=-2}$	$\Delta\epsilon_{s=-3}$	$\Delta\epsilon_{c=3}$	$\Delta\epsilon_{b=-3}$	$\Delta\epsilon_{s=-B}$
2	–	–	–	−0.075	−0.029	0.02	–	–	–	−0.075
3	0.000	0.034	0.07	–	–	–	−0.083	0.002	0.09	−0.082
4	–	–	–	−0.047	0.030	0.09	–	–	–	−0.13
5	−0.003	0.032	0.06	–	–	–	−0.060	0.035	0.12	−0.15
6	–	–	–	−0.044	0.025	0.09	–	–	–	−0.21
7	0.000	0.040	0.07	–	–	–	−0.042	0.068	0.15	−0.20
8	–	–	–	−0.039	0.023	0.03	–	–	–	−0.28

$$= \frac{|F|(|F|+2)}{8}\left(\frac{\mu_{F,B}-1}{\Theta_{F,B}\mu_{F,B}} - \frac{1}{\Theta_{T,B}}\right) - \frac{T_{\rm r}(T_{\rm r}+1)}{2\Theta_{T,B}} + T_{\rm r}|F|\frac{\mu_{F,B}-1}{4\Theta_{F,B}\mu_{F,B}}. \tag{35a}$$

At large $|F|$ this is approximately given by

$$E^{I^{\max}} - E^{I=0} \simeq \frac{\vec{I}_F^2}{2\Theta_{T,B}}(1 - 2c_{F,B}). \tag{35b}$$

At large B and F the isoscalar states have smaller energy if $c_{F,B} \leq 0.5$; see also Table 1.

Let us now consider the case corresponding to the bulk of "flavoured matter", i.e. $p = q = B = |F|$. Such "multilambda" states with isospin equal to zero have the following value of $\Delta\epsilon$ for $B \gg 1$:

$$\Delta\epsilon \simeq |F|\left[\omega_{F,1} - \omega_{F,B} + \frac{|F|+2}{8}\left(\frac{\mu_{F,B}-1}{\Theta_{F,B}\mu_{F,B}^2} - \frac{1}{\Theta_{T,B}}\right) - \frac{3(\mu_{F,1}-1)}{8\mu_{F,1}^2\Theta_{F,1}}\right]. \tag{36}$$

At large $|F|$ the sign of this expression depends on the sign of the difference $(\mu_{F,B}-1)/(\Theta_{F,B}\mu_{F,B}^2) - 1/\Theta_{T,B}$. To draw final conclusions we require the knowledge of the behaviour of the ratio $\Theta_{T,B}/\Theta_{F,B}$ at large B. For heavy flavours, c and b, $\mu_{F,B} \gg 1$ and the second term, (35b), is negative, unless $\Theta_{T,B} \sim \mu_{F,B}\Theta_{F,B}$ which is not realistic (we have usually $\mu_s \sim 3$, $\mu_c \sim 15$ and $\mu_b \sim 73$). So, for heavy flavours it is not possible to obtain, in this way, the bulk of flavoured matter as quantized coherent multiskyrmions. It should be kept in mind that for large $|F|$, say $|F| \geq 3$, the "rigid oscillator" model in its present form cannot be taken seriously: further terms in the expansion in $D^\dagger D$ should be taken into account in the Lagrangian. And other possibilities remain to be investigated, e.g. flavoured skyrmion crystals.

As in the $B = 1$ case [26], the absolute values of masses of multiskyrmions are controlled by the poorly known loop corrections to their classic mass, or the Casimir energy. And as has been done for the $B = 2$ states [18] the renormalization procedure is necessary to obtain physically reasonable values of these masses. As the binding energy of the deuteron is 30 MeV instead of the measured value 2.23 MeV we see that ~ 30 MeV characterises the uncertainty of our approach [17,18]. But this uncertainty cancels in the differences of binding energies presented in Tables 2 and 3.

5 Conclusions

Using rational map ansaetze as starting configurations we have calculated the static properties of bound skyrmions with baryon numbers up to 8. The excitation frequencies for different flavours – strangeness, charm and bottom – have been estimated using a rigid oscillator version of the bound state approach of the chiral soliton models. One notes that, in comparison with strangeness, this approach works even better for c and b flavours [20,21]. Our previous conclusion that BS with charm and bottom have more chance to be bound by strong interactions than strange BS [17] is reinforced by the present investigation. Estimates of the binding energy differences of flavoured and flavourless states have some uncertainty, of about few tens of MeV, but the tendency for charm and bottom to be more strongly bound than strangeness is very clear.

A natural question now arises as to how these results depend on the choice of the parameters of the model. A set of parameters, which has been used extensively in the literature, is, e.g. the set introduced in [9] where the masses of the nucleon and the Δ isobar have been fitted in the massless case and with the physical pion mass, respectively. In view of the large negative contribution of the loop corrections, or of the Casimir energy, we feel that this choice of parameters cannot be taken too seriously. However, the calculations show that our results hold for this choice too. The energies of the flavour excitations are somewhat smaller, however: for example, for strangeness $\omega_s = 255\,\mathrm{MeV}$ for $B = 1$ and $249\,\mathrm{MeV}$ for $B = 4$, if we take $F_K/F_\pi = 1.22$, $F_\pi = 108\,\mathrm{MeV}$. Similar changes take place for charm and bottom, and the conclusion that charmed or bottomed BS have good chances to be stable against strong interactions remains valid [17].

It should be kept in mind that corrections of the order of $1/N_c^2$ can lead to some modifications of our results. For example, the flavour moment of inertia changes [18] lead to

$$\Theta_F \to \Theta_F^{(0)} - D^\dagger D \frac{F_D^2 - F_\pi^2}{8} \int (1-c_f)(2-c_f)\mathrm{d}^3 r. \quad (37)$$

The decrease of the moment of inertia could lead to some increase of the zero mode quantum corrections.

The apparent drawback of our approach is that the motion of the system into the “strange”, “charm” or “bottom” directions has been considered independently from other motions. Consideration of the BS with “mixed” flavours is possible in principle, but its treatment would be more involved (see, e.g., [27] where the collective coordinate approach to the quantization of $SU(n)$ skyrmions has been investigated). It should be noted that we did not consider here the so-called H-particle and related topics. Within the chiral soliton approach the H-particle with $B = 2$ appears as a soliton on the $SO(3)$ subgroup of $SU(3)$ [11,12], is a $SU(3)$ singlet and is expected to be strongly bound; see also the discussion in [19].

Our results agree qualitatively with the results of [28] where the strangeness excitation frequencies have been calculated within the bound state approach. The difference is, however, in the behaviour of excitation frequencies: we have found that they decrease when the baryon number increases from $B = 1$ to 4, thus increasing the binding energy of the corresponding BS. This behaviour seems to be quite natural: there is an attraction between K, D or B meson field and a $B = 1$ nucleon, and the attraction of a meson by 2, 3 etc. nucleons is greater. At some value of B saturation takes place. Similar results hold for ordinary nuclei: the binding energy of a deuteron is $2.22\,\mathrm{MeV}$ only, for $B = 3$ it is about $8\,\mathrm{MeV}$, for $B = 4$ it is already $28\,\mathrm{MeV}$, and soon saturation takes place.

There is a further difference between the rigid oscillator variant of the bound state model we have used here and the collective coordinate approach of soliton models studied previously [10–12]. In the collective coordinate approach involving zero modes of solitons with a rigid or a soft rotator variant of the model, the masses of baryons are usually considerably greater than in the bound state approach when the Casimir energies are not taken into account [26,29]. One of the sources of this difference is the presence of a term of order N_c/Θ_F in the zero mode contribution to the rotation energy, which is absent in the bound state model. Recently, it was shown by Walliser, for the $B = 1$ sector within the $SU(3)$ symmetrical ($m_K = m_\pi$) variant of the Skyrme model [29], that this large contribution is cancelled almost completely by the kaonic 1-loop correction to the zero-point Casimir energy which is of the same order of magnitude, N_c^0 [29]. This correction has also recently been calculated within the bound state approach to the Skyrme model [30].

The charmed baryonic systems with $B = 3, 4$ were considered in [31] within a potential approach. The $B = 3$ systems were found to be very near the threshold and the $B = 4$ system was found to be stable with respect to the strong decay, with a binding energy of $\sim 10\,\mathrm{MeV}$.

Experimental searches for the baryonic systems with flavour different from u and d could shed more light on the dynamics of heavy flavours in systems with few baryons. The negative charge fragment seen in the NA52 CERN experiment [25] may be explained in our approach as a quantized $B = 7$ skyrmion with strangeness $S = -3$ or -4. The other possibility is $B = 6$ and $S = -6$ or -7. The value of the strangeness can be greater since the rigid oscillator version of the model we consider here overestimates the strangeness excitation energies.

The threshold for the charm production on a free nucleon is about $12\,\mathrm{GeV}$, and for the double charm it is $\sim 25.2\,\mathrm{GeV}$. For bottom, the threshold on a nucleon is $\sim 70\,\mathrm{GeV}$. However, for nuclei as targets the thresholds are much lower due to the two-step processes with mesons in intermediate states and due to the normal Fermi motion of nucleons inside the target nucleus (see, e.g., [32]). Therefore, the production of baryons or baryonic systems with charm and bottom should be feasible in proton accelerators with energies of several tens of GeV, as well as in heavy ions collisions.

Let us finish by adding that a shortened (and much less complete) version of this paper is available in [33]. The results obtained recently in [34] within the detailed version of the bound state approach are in fair agreement with ours, but the binding energies obtained in [34] are smaller than what we have found here. It should be noted that, different from [34], we have used the empirical values of flavour decay constants, taken into account the $1/N_c$ zero modes contributions to the energy of multibaryons and have considered only the difference of binding energies of flavoured BS and of the ground states where many of the uncertainties cancel out.

Acknowledgements. We are indebted to B.E. Stern for help in numerical calculations and to J. Madsen for informing us about the result of the NA52 experiment [25]. This work has been supported by the UK PPARC grant: PPA/V/S/1999/00004.

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